FAST WAVELET TRANSFORM IN MOTOR UNIT ACTION POTENTIAL ANALYSIS

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Abstract- Wavelet analysis is a new method for analyzing time and frequency contents of signals. We present the Fast Wavelet Transform (FWT) implemented using B-wavelets and non-circular convolutions in the analysis of Motor Unit Action Potentials (MUAPs). This method allows for the fast extraction of localized frequency components of MUAPs that may prove to be valuable in the early and accurate diagnosis of neuromuscular disorders.

I. INTRODUCTION

Structural reorganization of the motor unit, the smallest functional unit of muscle, takes place because of disorders affecting peripheral nerve and muscle. Motor unit morphology can be studied by recording its electrical activity, the procedure known as electromyography (EMG). When using a needle electrode, and at slight voluntary contraction, motor unit action potentials (MUAPs) are recorded. Features of MUAPs extracted in the time domain such as duration, amplitude, and the number of phases proved to be valuable in differentiating between muscle and nerve diseases [1]. On the other hand, it has been demonstrated that there is little to be gained by FFT based MUAP features [2]. The objective of this communication is to examine how the Fast Wavelet Transform (FWT) may be used to extract MUAP features that may be applied as a useful diagnostic tool. The potential of wavelet analysis in EMG signal processing has also recently been mentioned by [3].

II. THE FAST WAVELET TRANSFORM

The wavelet transform represents functions as coefficients of discrete translations and dilations of a mother wavelet function $\psi(x)$ (the discrete dilations and translations are represented by $\psi_{j,k}(x)$. If $\psi_{j,k}(x)$ is orthogonal to its discrete dilations and translations (represented by $\psi_{j'k'}(x)$ where $j'\neq j$ and $k'\neq k$), then the wavelet coefficients $\{d_k^j\}$ for a particular function $f(x)$ are calculated by taking inner products of $f(x)$ with the wavelet function $\psi_{j,k}(x)$ (in other words, $d_k^j = \langle f(x), \psi_{j,k}(x)\rangle$ where d_k^j is a "detail" coefficient [4]). On the other hand, if the wavelet function $\psi_{j,k}(x)$ is only orthogonal to its dilations, then we use a dual wavelet $\tilde{\psi}_{j,k}(x)$ to calculate the wavelet coefficients (which is now orthogonal to the discrete translations and dilations of $\psi_{j,k}(x)$). In this case, we call $\psi_{j,k}(x)$ a semi-orthogonal wavelet, and we represent all finite energy signals $f(x)$ by

$$f(x) = \sum_{j=-\infty}^{j=\infty}\sum_{k=-\infty}^{k=\infty} d_k^j \psi_{j,k}(x) = \sum_{j=-\infty}^{j=\infty}\sum_{k=-\infty}^{k=\infty} \langle f(x), \tilde{\psi}_{j,k}(x)\rangle \psi_{j,k}(x) \quad for\ j,k\in Z. \tag{1}$$

We define the discrete dilations and translations of $\psi(x)$ by

$$\psi_{j,k}(x) = 2^{\frac{j}{2}}\psi(2^j x-k). \tag{2}$$

In addition, the inner product is defined as:

$$\langle f(x), \tilde{\psi}_{j,k}(x)\rangle = \int_{-\infty}^{+\infty} f(x)\tilde{\psi}^*_{j,k}(x)\,dx. \tag{3}$$

In traditional signal analysis, a window function is defined such that by taking inner products of a signal with the window function (as in (3)), the fourier transform of the inner product provides an accurate measure of the spectral content of the signal for a certain time interval. In contrast, the wavelet function $\psi(x)$ is defined to posses oscillation so that it localizes the signal and its fourier transform directly [4]. Therefore, the amplitude of the detail coefficients $\{d_k^j\}$ are used as a measure of frequency content (as indexed by j) for a particular time interval (as indexed by k).

In order to efficiently calculate the wavelet transform (as defined by (1)), we need to break the infinite sums over j and k. First, we observe that for $j=0$ (resolution level = 0), the argument of the wavelet function in (2) becomes $(x-k)$ which varies over the integers for x being an integer. Thus, if we consider our signal samples as an array of samples (having an integer index), we conclude that we need to pick $j=0$ to be the resolution level of our signal in order to match the arguments of the functions in the inner product, see (3). In addition, the length of the array of signal samples restricts k to a finite interval. Next, we wish to calculate the infinite sum over j on the left (for $j=-n-1$ to $-\infty$) by using a scaling function $\varphi(x)$ such that

$$\sum_k c_k^{-n}\phi_{-n,k}(x) = \sum_{j=-\infty}^{j=-n-1}\sum_k d_k^j\psi_{j,k}(x). \tag{4}$$

Naturally, there is no guarantee that a scaling function $\varphi(x)$ can be found so that we can write (4). In fact, we can only write (4) when the wavelet function $\psi(x)$ is defined in terms of $\varphi(x)$ which (the scaling function) generates a Multi-Resolution Analysis [4]. Thus, let us assume that (4) is possible and plug (4) into (1) to get

$$f(x) = \sum_{j=0}^{j=-n}\sum_k d_k^j\psi_{j,k}(x) + \sum_k c_k^{-n}\phi_{-n,k}(x). \tag{5}$$

Thus by (5), we have reduced the problem of calculating the FWT and the Inverse Wavelet Transform (IWT) as a problem of determining $\{d_k^j\}$ ($j=0$ to $-n$) and $\{c_k^{-n}\}$. As indicated by (5), we can calculate as many resolution levels as we want (indicated by ($j=0$ to $-n$), and still be able to perform perfect reconstruction. In addition, observe that if we consider the wavelet coefficients $\{d_k^j\}$ as representing the "detail" or time-

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frequency content extracted from the signal, then the $\{c_k^j\}$ represent the "smooth" coefficients which approximate the signal (after the "details" have been removed). In addition, due to the Multi-Resolution Analysis generated by the scaling function, we can calculate $\{d_k^j\}$ and $\{c_k^{-n}\}$ by using the following recursive formulas [4]

$$c^{j-1} = \{h_k * c_k^{\,j}\}\downarrow 2 \tag{6}$$

$$d^{j-1} = \{g_k * c_k^{\,j}\}\downarrow 2 \tag{7}$$

Thus, using (6) and (7) alone we can calculate the FWT (as indicated by (5)), if we knew how to calculate $\{c_k^0\}$. If $\{c_k^0\}$ was given, then the rest of the coefficients could be calculated by simply convolving the $\{c_k^0\}$ coefficients with the $\{h_k\}$ and $\{g_k\}$ filters and downsampling (throwing away every other point as indicated by (6) and (7). We can take the $\{c_k^0\}$ coefficients to simply be our signal samples, or we can use projection operators that take the signal samples and generate the $\{c_k^0\}$ coefficients [4]. In our case, we use the B-wavelets for which we can use the well-known B-splines projection operators for calculating $\{c_k^0\}$ (the B-splines are the scaling functions $\varphi(x)$). Thus, we used the Quasi-Interpolator given in [4] to get $\{c_k^1\}$, and then use (6) to calculate $\{c_k^0\}$.

III. ANALYSIS OF RESULTS

MUAPs were recorded from the biceps brachii muscle, band-pass filtered at 2 Hz to 10 KHz and sampled at 20 KHz with 12 bits resolution. The results of the FWT applied to a MUAP are shown in Fig. 1. Since the beginning and ending of a MUAP is determined by a rise or fall of the signal amplitude from the zero baseline, we assumed that the signal was zero outside the interval of our signal samples. Thus, we implemented (6) and (7) via non-circular convolutions. This approach provides us with a better MUAP signal approximation.

Note that $\{d_k^j\}$ coefficients for high j represent time-frequency contents of high frequencies. Similarly, low j values correspond to time-frequency contents of low frequencies. In addition, we observe that the signal approximation $\{c_k^j\}$ coefficients get decimated by two in every stage. Thus, a localized high frequency component becomes "critically sampled" when in the time interval that it occurs, there are no longer enough points in the approximation coefficients $\{c_k^j\}$ to represent it. This is very vividly shown from the c^{-1} to c^{-2} and d^{-2} projections in Fig. 1. The high frequencies in the MUAP main spike are seen removed from the c^{-1} coefficients and appear in the d^{-2} projection. This is demonstrated by the tenfold increase in the d^{-2} wavelet coefficients present in the region within the MUAP main spike. Similarly, for the next projection the high frequencies present in the region of the main spike are seen removed from c^{-2} and projected to d^{-3}.

In conclusion, the FWT makes possible decomposition of MUAPs into highly localized time-frequency components that was not possible before. This decomposition should be further explored to see if it provides additional diagnostic information that may have not been apparent in current MUAP analysis techniques.

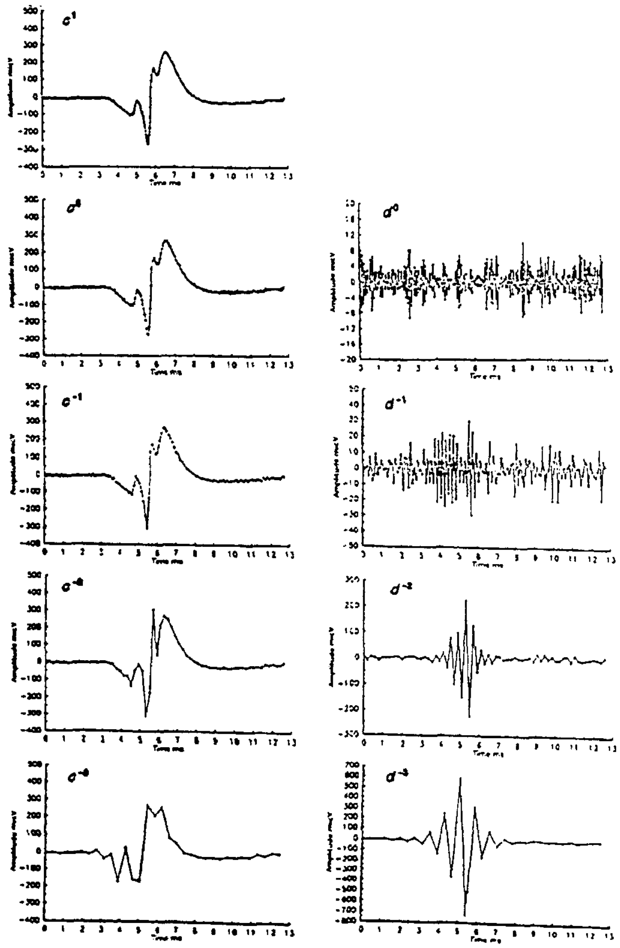

Fig. 1 *MUAP wavelet decomposition.* The FWT results are shown for the low-pass filter output coefficients c^j and the band-pass output coefficients d^j. The coefficients have been normalized to the scale of the original signal.

REFERENCES

[1] E. Stalberg, et al, "Quantitative Analysis of individual motor unit potentials: A proposition for standardised terminology and criteria for measurement," *J. Clin. Neuroph.*, 3(4), pp. 313-348, 1986.

[2] D. Stashuk, and R. Naphan, "Classification performance of different motor unit action potential feature space representations: a simulation study," *Proceedings of the 12th IEEE EMBS Conference*, pp. 2225-2226, 1990.

[3] K.C. McGill, "EMG signal-processing techniques for motor-unit analysis," *Abstracts of the IX Intern. Congr. of EMG and Clin. Neuroph.*, Jerusalem, Israel, pp. 64, 1992.

[4] C.K. Chui (Ed), *Wavelet analysis and its applications*, Academic Press, London, vol. I, 1992.